



Regular Expressions

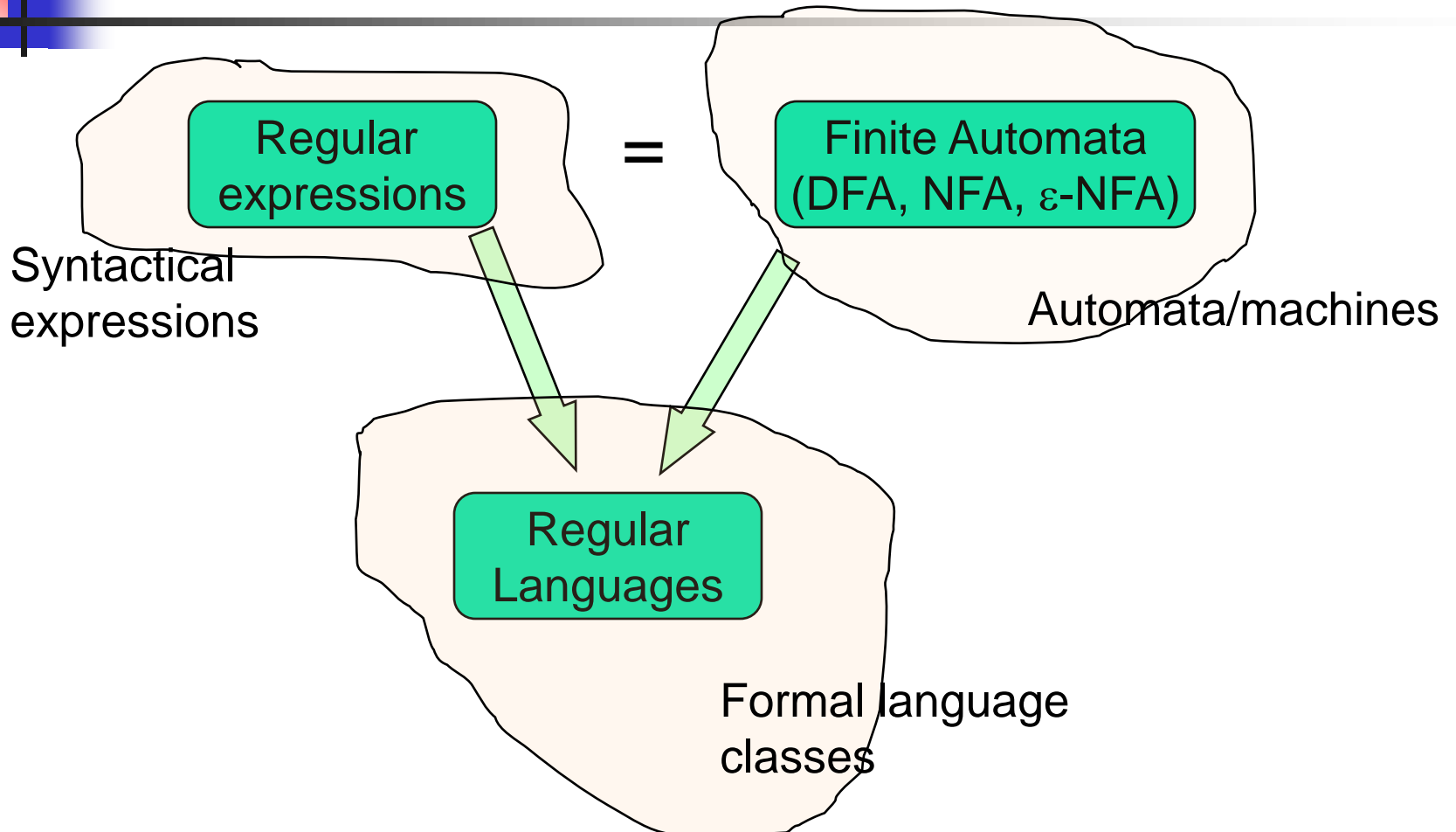
Reading: Chapter 3



Regular Expressions vs. Finite Automata

- Offers a declarative way to express the pattern of any string we want to accept
 - E.g., $01^* + 10^*$
- Automata => more machine-like
 - < input: string , output: [accept/reject] >
- Regular expressions => more program syntax-like
- Unix environments heavily use regular expressions
 - E.g., bash shell, grep, vi & other editors, sed
- Perl scripting – good for string processing
- Lexical analyzers such as Lex or Flex

Regular Expressions





Language Operators

- Union of two languages:
 - $L \cup M$ = all strings that are either in L or M
 - Note: A union of two languages produces a third language

- Concatenation of two languages:
 - $L . M$ = all strings that are of the form xy
s.t., $x \in L$ and $y \in M$
 - The *dot* operator is usually omitted
 - i.e., LM is same as $L.M$

“i” here refers to how many strings to concatenate from the parent language L to produce strings in the language L^i

Kleene Closure (the * operator)

- Kleene Closure of a given language L:
 - $L^0 = \{\epsilon\}$
 - $L^1 = \{w \mid \text{for some } w \in L\}$
 - $L^2 = \{w_1w_2 \mid w_1 \in L, w_2 \in L \text{ (duplicates allowed)}\}$
 - $L^i = \{w_1w_2\dots w_i \mid \text{all } w\text{'s chosen are } \in L \text{ (duplicates allowed)}\}$
 - (Note: the choice of each w_i is independent)
 - $L^* = \bigcup_{i \geq 0} L^i$ (arbitrary number of concatenations)

Example:

- Let $L = \{1, 00\}$
 - $L^0 = \{\epsilon\}$
 - $L^1 = \{1, 00\}$
 - $L^2 = \{11, 100, 001, 0000\}$
 - $L^3 = \{111, 1100, 1001, 10000, 000000, 00001, 00100, 00111\}$
 - $L^* = L^0 \cup L^1 \cup L^2 \cup \dots$



Kleene Closure (special notes)

- L^* is an infinite set iff $|L| \geq 1$ and $L \neq \{\varepsilon\}$ **Why?**
- If $L = \{\varepsilon\}$, then $L^* = \{\varepsilon\}$ **Why?**
- If $L = \Phi$, then $L^* = \{\varepsilon\}$ **Why?**

Σ^* denotes the set of all words over an alphabet Σ

- Therefore, an abbreviated way of saying there is an arbitrary language L over an alphabet Σ is:
 - $L \subseteq \Sigma^*$



Building Regular Expressions

- Let E be a regular expression and the language represented by E is $L(E)$
- Then:
 - $(E) = E$
 - $L(E + F) = L(E) \cup L(F)$
 - $L(E F) = L(E) L(F)$
 - $L(E^*) = (L(E))^*$

Example: how to use these regular expression properties and language operators?

- $L = \{ w \mid w \text{ is a binary string which does not contain two consecutive 0s or two consecutive 1s anywhere} \}$
 - E.g., $w = 01010101$ is in L , while $w = 10010$ is not in L
- Goal: Build a regular expression for L
- Four cases for w :
 - Case A: w starts with 0 and $|w|$ is even
 - Case B: w starts with 1 and $|w|$ is even
 - Case C: w starts with 0 and $|w|$ is odd
 - Case D: w starts with 1 and $|w|$ is odd
- Regular expression for the four cases:
 - Case A: $(01)^*$
 - Case B: $(10)^*$
 - Case C: $0(10)^*$
 - Case D: $1(01)^*$
- Since L is the union of all 4 cases:
 - Reg Exp for $L = (01)^* + (10)^* + 0(10)^* + 1(01)^*$
- If we introduce ε then the regular expression can be simplified to:
 - Reg Exp for $L = (\varepsilon + 1)(01)^*(\varepsilon + 0)$



Precedence of Operators

- Highest to lowest

- * operator (star)
- . (concatenation)
- + operator

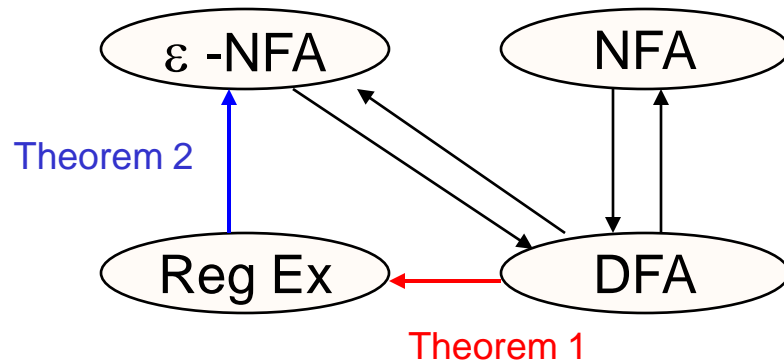
- Example:

- $01^* + 1 = (0 \cdot ((1)^*)) + 1$

Finite Automata (FA) & Regular Expressions (Reg Ex)

- To show that they are interchangeable, consider the following theorems:
 - Theorem 1: For every DFA A there exists a regular expression R such that $L(R)=L(A)$
 - Theorem 2: For every regular expression R there exists an ε -NFA E such that $L(E)=L(R)$

Proofs
in the book



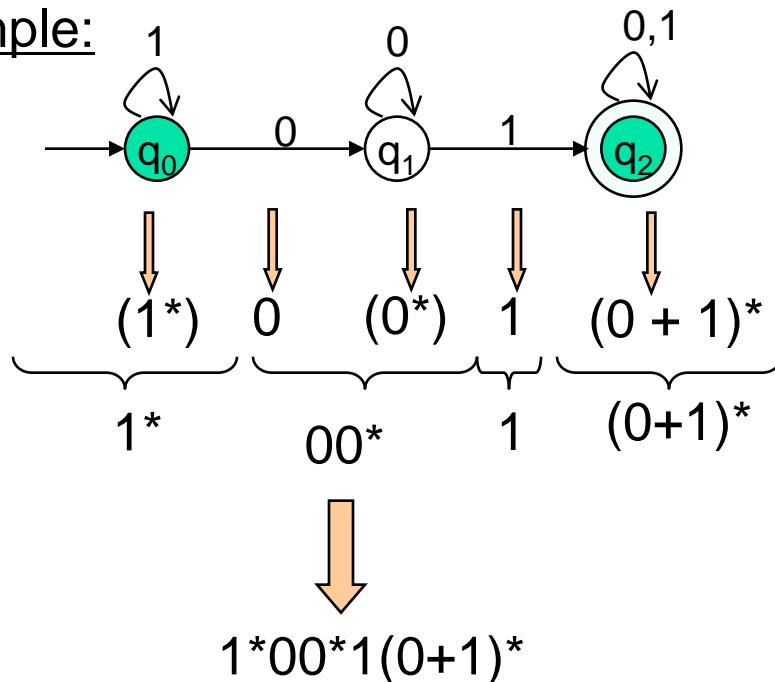
Kleene Theorem



DFA to RE construction

Informally, trace all distinct paths (traversing cycles only once) from the start state to *each of the* final states and enumerate all the expressions along the way

Example:



Q) What is the language?

Reg Ex

Theorem 2

ϵ -NFA

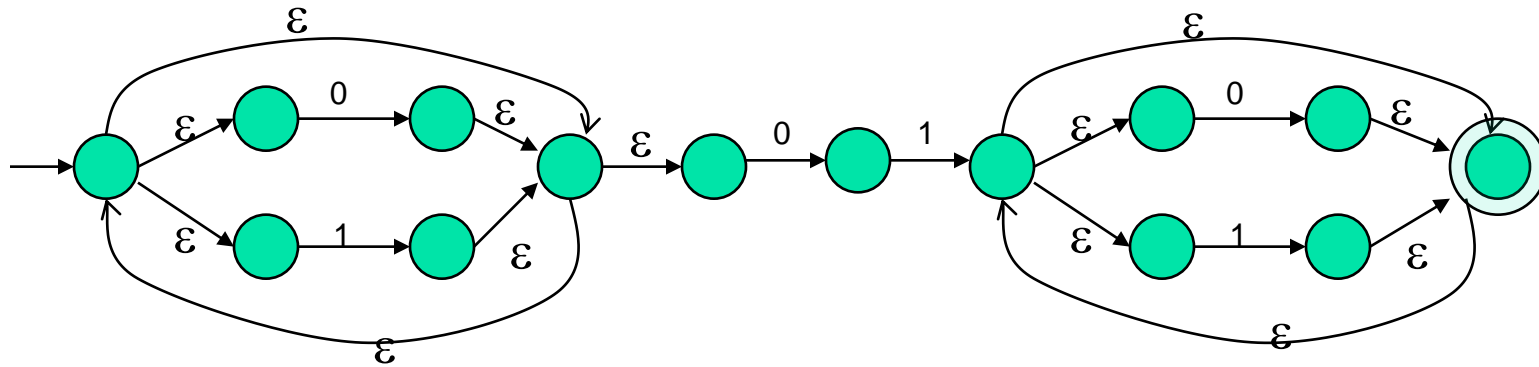
RE to ϵ -NFA construction

Example: $(0+1)^*01(0+1)^*$

$(0+1)^*$

01

$(0+1)^*$



Algebraic Laws of Regular Expressions

- Commutative:
 - $E+F = F+E$
- Associative:
 - $(E+F)+G = E+(F+G)$
 - $(EF)G = E(FG)$
- Identity:
 - $E+\Phi = E$
 - $\varepsilon E = E \varepsilon = E$
- Annihilator:
 - $\Phi E = E\Phi = \Phi$



Algebraic Laws...

- Distributive:
 - $E(F+G) = EF + EG$
 - $(F+G)E = FE+GE$
- Idempotent: $E + E = E$
- Involving Kleene closures:
 - $(E^*)^* = E^*$
 - $\Phi^* = \varepsilon$
 - $\varepsilon^* = \varepsilon$
 - $E^+ = EE^*$
 - $E? = \varepsilon + E$



True or False?

Let R and S be two regular expressions. Then:

1. $((R^*)^*)^* = R^*$?

2. $(R+S)^* = R^* + S^*$?

3. $(RS + R)^* RS = (RR^*S)^*$?